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The REX Phase Jumps: Identification and Correction

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1. Introduction

In the REX data downlinked from the 2007 Annual Checkout of the New Horizons Spacecraft, (an event otherwise known as ACO-7), Dave Hinson noticed an anomaly in the phase of the Uplink signal. Before describing the anomaly, I will define for you the phase of the Uplink signal and show you how to obtain it.

The Uplink is an X-band electromagnetic (EM) wave, transmitted unmodulated with a power of 20 kW from one of NASA's Deep Space Network's tracking stations. The term X-band means the frequency of the EM wave is between 8 GHz, to 12 GHz, and in this case the Uplink EM wave's frequency is 7.182 GHz. The Uplink is received in the New Horizons Spacecraft's X-band Receiver in the Telecom System, and shifted via heterodyning, into a 1.25 kHz bandwidth, centered at 0 Hz, in the REX instrument, where the Uplink is sampled simultaneously in two 'channels' called in-phase and quadrature (otherwise knows as I and Q).

In the REX instrument, the I and Q channels are sampled to 16-bits at a rate of 1250 samples per 1.024 seconds. The I and Q channels are almost identical, where the Q channel can be thought of conceptually as delayed in time, such that for each sample, the I and Q pair are a point in a two dimensional with I the x-axis, and Q, the y-axis. As such, each I and Q pair is a sample of a 'complex' amplitude, where the term 'complex' refers to an analytic representation with the I (or x-axis) the 'real' and Q (the y-axis) 'imaginary' components of the samples. In this context, the uplink samples in the REX band are represented as,

$$\begin{aligned} v_n &= a_o e^{2\pi i f_o t_n + \varphi_o} \\ &= a_o\left(\cos\left(2\pi f_o t_n + \varphi_o\right) + i\sin\left(2\pi f_o t_n + \varphi_o\right)\right) \\ &= v_n^{real} + v_n^{imag} \\ v_n^{real} &= a_o \cos\left(2\pi f_o t_n + \varphi_o\right) \\ v_n^{imag} &= a_o \sin\left(2\pi f_o t_n + \varphi_o\right) \end{aligned} \tag{1}$$

where: v_n = ‘the complex’ samples, f_o = the frequency in the REX band,

a_o = amplitude of the signal, φ_o = starting phase, t_n = sample time

The term ‘phase’ thus refers to the argument of the cosine and/or sine function, and in particular,

$$\varphi_n = 2\pi f_o t_n + \varphi_o$$

is the ‘phase’ of each sample. Note, in this case the phase is a linearly increasing function of the sample time, where the linear slope is the frequency f_o. Typically, this frequency is estimated, and removed from the phase such that the phase progression is no longer a linear function of time. Often the Uplink frequency is not quite constant and additional compensation is applied. But for illustration and for constant frequency, if *fest*, is the estimated frequency, then frequency compensation via multiplication becomes,

$$\begin{aligned} v_n^{comp} &= a_o e^{2\pi i f_o t_n + \varphi_o} e^{-2\pi i f_{est} t_n} \\ &= a_o e^{2\pi i (f_o - f_{est}) t_n + \varphi_o} \end{aligned} \tag{2}$$

The phase samples in this case are now,

$$\begin{aligned} \varphi_n^{comp} &= 2\pi (f_o - f_{est}) t_n + \varphi_o \\ &\cong \varphi_o \text{, a constant} \end{aligned}$$

Ideally, assuming the estimate of the Uplink’s frequency is exact, then the phase samples of the Uplink are all constant. Even for a constant frequency, the situation is not ideal, principally because the samples are perturbed by noise inherent in the thermal noise background and in the X-band receiver. This noise manifests as additive random fluctuations of the sample values. These perturbations can be represented as additive noise in Equations (1), and (2), where the noise samples are from a zero-mean, normal distribution whose standard deviation (STD), is proportional to the noise power:

$$
\begin{aligned}
v'_n &= a_o e^{2\pi i f_o t_n + \varphi_o} + v_n^{noise} \\
v_n^{compN} &= v'_n e^{-2\pi i f_{est} t_n} \\
&= v_n e^{-2\pi i f_{est} t_n} + v_n^{noise} e^{-2\pi i f_{est} t_n} \\
&= v_n^{comp} + v_n^{noise}
\end{aligned}
\tag{3}
$$

The phase of a signal perturbed by noise, in the limit where the noise amplitude is small compared to the signal amplitude, is obtained by taking the natural logarithm of the frequency-compensated samples. Using the approximation that the natural logarithm of 1 + x, is approximately x, the phase of the Uplink signal perturbed by noise is then,

$$
\begin{aligned}
\varphi_n^{compN} &= \ln\left(v_n^{compN} / a_o \right) \\
&= \ln\left((v_n^{comp} + v_n^{noise}) / a_o \right) \\
&\cong \varphi_o + \varphi_n^{noise}
\end{aligned}
\tag{4}
$$

Thus for signals much stronger than the background noise, i.e. where $a_o >> |v^{noise}|$, the frequency compensated phase samples are a sequence of random values, distributed according to the random distribution from which the noise itself resides. Typically, for EM waves in a radio receiver, the noise distribution is dominated by a simple, zero-mean Gaussian whose standard deviation (STD) is proportional to the RMS of the noise power.

For the vast majority of phase samples of the Uplink in REX, the phase appears as a random walk, with both short term fluctuations characterized by a STD much smaller than the signal amplitude, but with a mean value that wanders according to correlation lengths characteristic of a random walk’s 1/f nature. However, every so often, on the order of 200 seconds or so, the phase jumps abruptly, only on

Side-A, and only for a little while, and then returns to where it would have been if it had not jumped at all. This anomaly occurs only on Side-A of REX (i.e. the side dedicated to the Right Hand Circular Polarization (RCP) signal), and never on Side-B (the side dedicated to the Left Hand Circular Polarization (LCP) signal). Further, the interval between phase jumps is neither exact nor periodic, but has been ~200 seconds on average.

2. Identification

A good example of the phase jumps is included here in Figures 2a, and 2b. The principal characteristics of the phase jumps indicated in these figures are (a) the phase jumps occur in pairs with first a displacement followed some samples later by the opposite displacement, (b) the phase jumps are always nearly 0.1 radian in size, and (c) between the pair of phase jumps, the displacement of the phase sequence is always a constant. It's as if a section of the phase has been sliced out of the phase sequence and shifted upward or downward, by ~0.1 radian. Thus, the phase jumps are easily identified, easily located, and easily corrected.

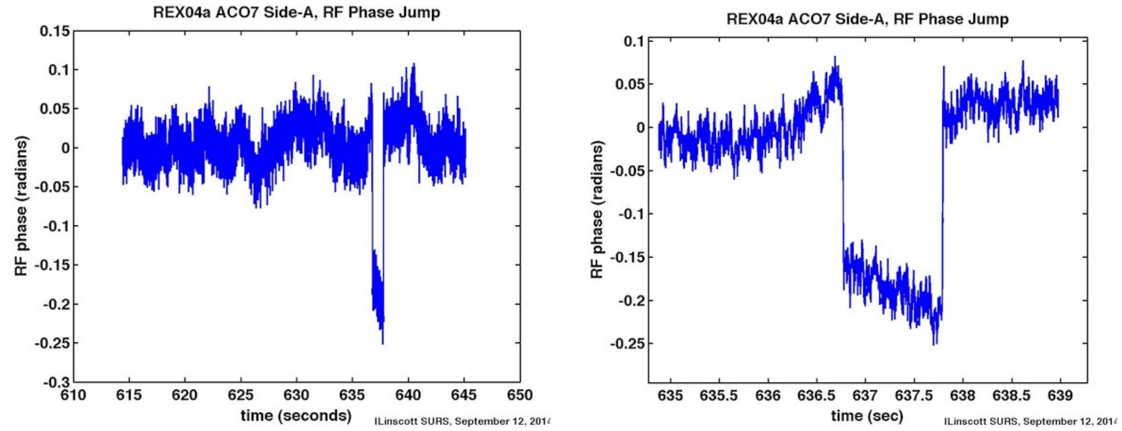

Figure 2. Example Phase Jump in REX Side-A. Fig. 2.a (left hand figure), Phase sequence of Uplink signal acquired during ACO-7. Phase has an initial offset of zero mean. Fig. 2.b (right hand figure), zoomed-in view of Fig. 2.a.

3. Correction

These phase jumps, or more precisely, phase discontinuities, occur within a single sample, i.e. sample φ_n, appears normal, and φ_{n+1}, is displaced by ~0.1 radian. This displacement continues for a finite number of samples, e.g. K samples, with sample φ_{n+K}, still displaced, but sample φ_{n+K+1}, reset back to the undisplaced sequence. The correction for the phase jump (or discontinuity), is first to identify the displaced samples, i.e. find n and K, and the displacement amount, e.g. 0.1 radian, and then to shift the K samples in value by the displaced amount. Although entirely empirical, this method of correction has been effective and exact for all the instances of phase discontinuity to date.

Applying this method of correction to the example in Figure 2, produces the corrected phase sequence shown in Figure 3. The effectiveness of this correction is validated in the restoration to continuity of the corrected Uplink waveform. Although a 0.1 radian phase shift is barely evident in the waveform (see the effect for example in Figure 4), a comparison of the Fourier transforms of the impacted samples of the waveform with the phase shift corrected waveform, as in figure 5, reveals that the discontinuity related artifacts have been suppressed. Note for example, the reduction in the close-in side-response in the spectrum near the 100 Hz line for the Uplink signal's frequency. The frequencies nearest the 100 Hz line are suppressed ~5 dB, a significant artifact reduction, particularly for any estimate that would be derived from the behavior of the 100 Hz signal.

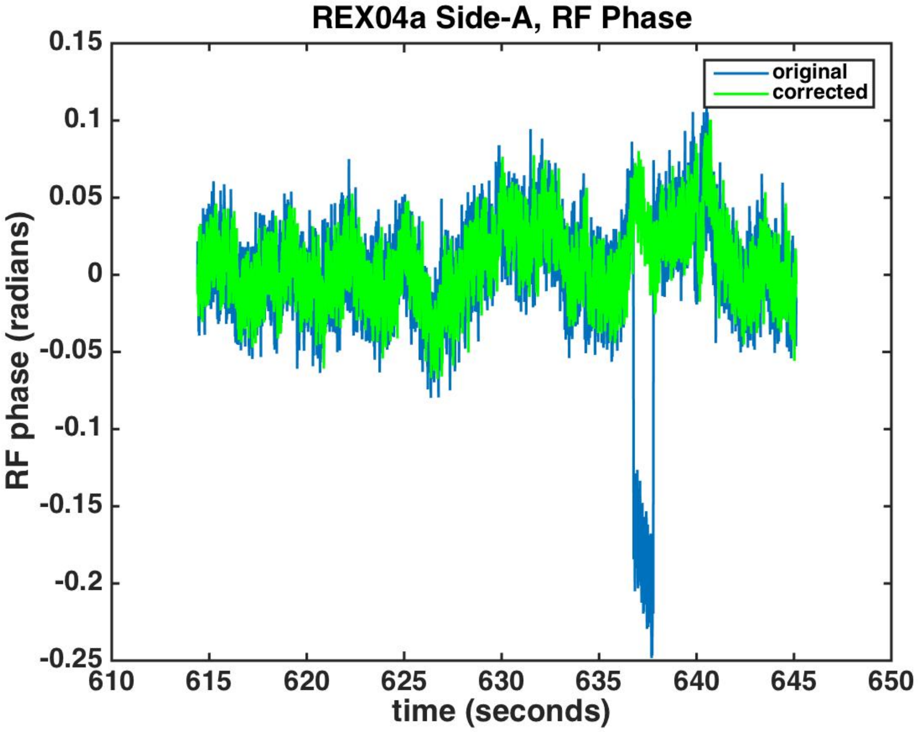

Figure 3. Original and Corrected Phase sequences. The blue trace is the original phase sequence of the portion of the Uplink signal containing the phase jump (or discontinuity), as shown in Figure 2. A single offset was identified from the discontinuity on the left-hand side of the jump, and used to correct the discontinuity by adding the offset to the phase within the interval of the jump. The green trace is the corrected phase produced by this method. [A slight scale reduction was imposed on the corrected phases for purposes of improving the illustration.]

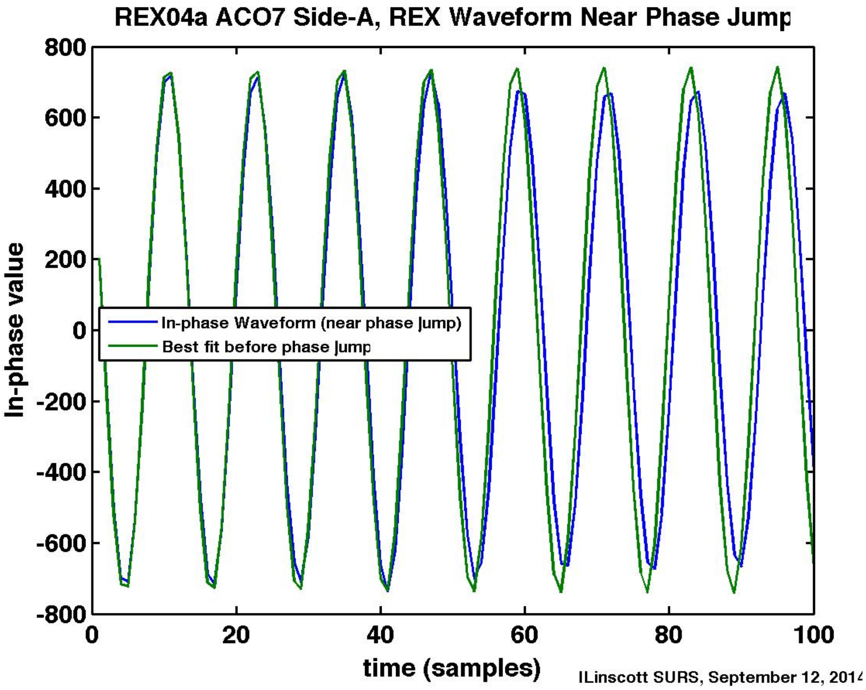

Figure 4. Uplink Waveform in REX Side-A. Phase jump is at sample no. 50. Blue trace is the Uplink waveform of the original samples interpolated between samples. The green trace is a best fit of the frequency, amplitude and phase offset using the first 50 samples, and then extending the fit functionally to the remaining samples. [Note that a time shift is functionally equivalent to a shift in phase.]

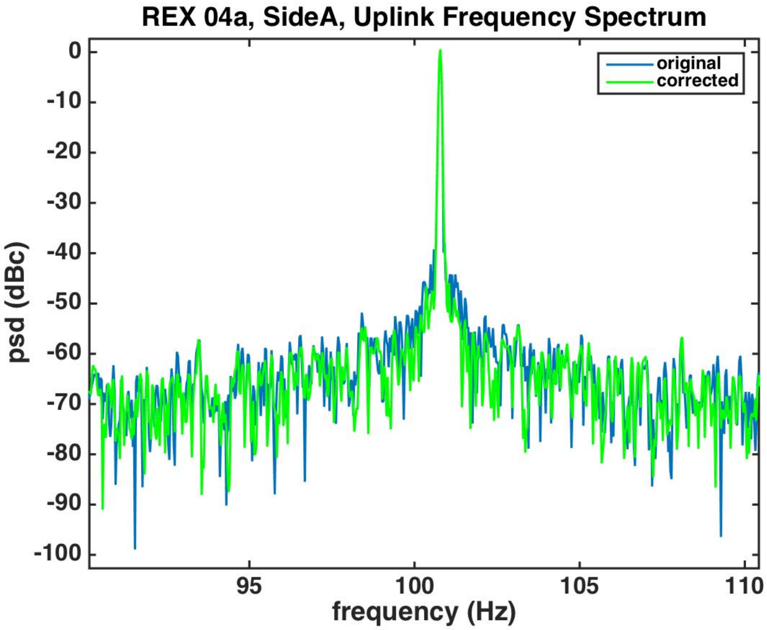

Figure 5. Portion of the Spectrum of the Uplink Signal. The blue trace is the spectrum of the original waveform, and the green trace is the

spectrum of the phase corrected waveform. Note: frequencies close to the 100 Hz line are suppressed.